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Coupled excitations in the compressible easy-plane Heisenberg chain

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Abstract. We investigate the solitonic excitations in the easy-plane compressible ferromagnetic Heisenberg chain using a coherent states formalism. It is shown that the existence of out-of-easy-plane excursions is compulsory for the excitations of the elastic modes. The shapes of the solitons are shown to be strongly dependent on the values of the parameters of the model.

Model substances representing magnetic chains have been investigated in detail by measuring neutron scattering cross sections [1] and by several other experimental techniques. The results reveal the presence of non-linear excitations of the magnetic degrees of freedom which are as elementary as the usual linear modes. A particular system which theories and experiments have studied intensely is the easy-plane linear-chain magnet $CsNiF_3$ in which the interactions between the spins are ferromagnetic; to compare theory and experiments many approximations has been performed, in particular that involving a sine–Gordon mapping [2]; of course this makes comparisons restricted and induces to consider more complex theoretical models, as those involving new and diverse degrees of freedom; for example models bringing magnetic and elastic excitations. The models are mainly based on the Heisenberg Hamiltonian of a chain of magnetic ions distributed along an axis and acted by strong anisotropies along some predetermined directions. Coupled excitations may then exist on different ground states.

The existence of coupled magnetic and elastic non-linear modes in the compressible ferromagnetic Heisenberg chain has been extensively demonstrated for the case of uniaxial anisotropy [3].

Our interest is to investigate the existence of these coupled excitations in the compressible ferromagnetic Heisenberg chain with easy-plane anisotropy in the presence of an external magnetic field parallel to that plane. The Hamiltonian appropriate for this system is

$$\mathcal{H} = \mathcal{H}_{\mathrm{mag}} + \mathcal{H}_{\mathrm{ph}} + \mathcal{H}_{\mathrm{int}} \tag{1}$$

where $\mathcal{H}_{\mathrm{mag}}$ represents the magnetic degrees of freedom and has the form

$$\mathcal{H}_{\mathrm{mag}} = -J \sum_{n,\delta=\pm a}^{N} \boldsymbol{S}_n \cdot \boldsymbol{S}_{n+\delta} + D \sum_{n=1}^{N} (S_n^z)^2 - h \sum_{n=1}^{N} S_n^x \tag{2}$$

with S_n the spin on site n and $J > 0$; $D > 0$, $h \equiv g\mu_B H$.

The elastic modes are represented by $\mathcal{H}_{\text{ph}}$

$$\mathcal{H}_{\text{ph}} = \sum_{n=1}^{N} \frac{p_n^2}{2m} + \frac{k}{2} \sum_{n,\delta=\pm a}^{N} (q_{n+\delta} - q_n)^2 \tag{3}$$

where the p_n are the momenta and q_n the longitudinal displacements from the equilibrium positions. m is the mass of the ions and k is the elastic constant.

The term $\mathcal{H}_{\text{int}}$ represents the interaction between magnetic and elastic modes; to first order this term takes the form

$$\mathcal{H}_{\text{int}} = -\lambda \sum_{n,\delta=\pm a}^{N} (q_{n+\delta} - q_n) S_n \cdot S_{n+\delta} \tag{4}$$

where $\lambda \equiv \partial J/\partial z$ is the magnetostriction parameter and a represents the lattice spacing.

Proceeding as in [3] we take the continuous form of the Hamiltonian (1)

$$\begin{aligned}\mathcal{H}_{\text{mag}} = -\frac{J}{2} \int \mathrm{d}z \left(\frac{1}{2}(S^{\dagger}(z)S^{-\prime\prime}(z) + S^{-}(z)S^{\dagger\prime\prime}(z)\right) + S^{z}(z)S^{z\prime\prime}(z)) \\ + D \int \mathrm{d}z\, (S^{x}(z))^2 - h \int \mathrm{d}z\, S^{x}(z) - \frac{1}{2}NJS^2\end{aligned} \tag{5}$$

$$\mathcal{H}_{\text{ph}} = \frac{1}{2m} \int \mathrm{d}z\, (p(z))^2 + k \int \mathrm{d}z\, (q'(z))^2 \tag{6}$$

$$\mathcal{H}_{\text{int}} = -2\lambda \int \mathrm{d}z \left(\frac{1}{2}(S^{\dagger}(z)S^{-\prime}(z) + S^{-}(z)S^{\dagger\prime}(z)\right) + S^{z}(z)S^{z\prime}(z)) - \lambda S^2 \int \mathrm{d}z\, q''(z) \tag{7}$$

Next we transform this Hamiltonian, taking

$$p = \mathrm{i}\sqrt{m\omega_0/2}\,(c^{\dagger} - c) \qquad q = \frac{1}{\sqrt{2m\omega_0}}\,(c^{\dagger} + c)$$

where $\omega_0 = \sqrt{k/m}$; and, following [4], making use of the Schwinger representation

$$\begin{aligned} S^{\dagger}(z) &= a^{\dagger}(z)b(z) \\ S^{-}(z) &= b^{\dagger}(z)a(z) \\ S^{z}(z) &= \tfrac{1}{2}[a^{\dagger}(z)a(z) - b^{\dagger}(z)b(z)] \end{aligned} \tag{8}$$

where $a^{\dagger}(z)$, $a(z)$, $b^{\dagger}(z)$, $b(z)$ are simple harmonic oscillator bosonic operators.

In the Heisenberg picture the dynamic of the above operators is coupled through the following three equations

$$\mathrm{i}\dot{a}(z,t) = [a(z,t), \mathcal{H}] \qquad \mathrm{i}\dot{b}(z,t) = [b(z,t), \mathcal{H}] \qquad \mathrm{i}\dot{c}(z,t) = [c(z,t), \mathcal{H}] \tag{9}$$

where the dot stands for $\partial/\partial t$ and $\hbar = 1$.

Substituting (8) into (7) gives

$$
\begin{aligned}
i\dot{a} = {} & J(-\tfrac{3}{8}a'' - \tfrac{1}{4}a^\dagger a'' a - a' b'^\dagger b - \tfrac{1}{4}b''^\dagger b a - \tfrac{1}{2}b^\dagger b a'') \\
& + J(\tfrac{1}{2}a b'^\dagger b' + \tfrac{1}{4}a b^\dagger b'' - \tfrac{1}{4}a''^\dagger a^2 - \tfrac{1}{2}a'^\dagger a' a) \\
& + \tfrac{1}{2}D\,(a^\dagger a - b^\dagger b + \tfrac{1}{2})\,a - \tfrac{1}{2}\,h\,b \\
& + \frac{\lambda}{2\sqrt{2m\omega_0}}[\,(c^{\dagger\prime\prime} + c'')(a^\dagger a a + b^\dagger b b) - 3(c^{\dagger\prime} + c')a'\,]
\end{aligned} \tag{10}
$$

$$
\begin{aligned}
i\dot{b} = {} & J(-\tfrac{3}{8}b'' - \tfrac{1}{4}b^\dagger b'' b - b' a'^\dagger a - \tfrac{1}{4}a''^\dagger a b - \tfrac{1}{2}a^\dagger a b'') \\
& + J(\tfrac{1}{2}b a'^\dagger a' + \tfrac{1}{4}b a^\dagger a'' - \tfrac{1}{4}b''^\dagger b^2 - \tfrac{1}{2}b'^\dagger b' b) \\
& + \tfrac{1}{2}D\,(b^\dagger b - a^\dagger a + \tfrac{1}{2})\,b - \tfrac{1}{2}\,h\,a \\
& + \frac{\lambda}{2\sqrt{2m\omega_0}}[\,(c^{\dagger\prime\prime} + c'')(b^\dagger b b + a^\dagger a a) - 3(c^{\dagger\prime} + c')b'\,]
\end{aligned} \tag{11}
$$

$$
\begin{aligned}
i\dot{c} = {} & -\frac{\omega_0}{2}(c^\dagger - c) - \frac{k}{m\omega_0}(c^{\dagger\prime\prime} + c'') \\
& + \frac{\lambda}{2\sqrt{2m\omega_0}}[\,a^\dagger b^{\dagger\prime\prime} b a + b^\dagger a^\dagger a b'' + a^\dagger b^\dagger b a'' + b^\dagger a^{\dagger\prime\prime} a b \\
& + a^{\dagger\prime} a^{\dagger\prime} a a + a^\dagger a^\dagger a' a' + a^\dagger a^{\dagger\prime\prime} a a + a^\dagger a^\dagger a a'' \\
& + b^{\dagger\prime} b^{\dagger\prime} b b + b^\dagger b^\dagger b' b' + b^\dagger b^{\dagger\prime\prime} b b + b^\dagger b^\dagger b b''\,] \\
& + \frac{\lambda}{\sqrt{2m\omega_0}}[\,a^{\dagger\prime} b^{\dagger\prime} b a + a^\dagger b^\dagger b' a' + a^\dagger b^{\dagger\prime} b a' + b^\dagger a^{\dagger\prime} a b' \\
& + a^\dagger b^{\dagger\prime} b' a + a^{\dagger\prime} b^\dagger b a' + 4(a^\dagger a^{\dagger\prime} a' a + b^\dagger b^{\dagger\prime} b' b) \\
& + 3(a^{\dagger\prime} a' + a^\dagger a'' + b^{\dagger\prime} b' + b^\dagger b'')\,]
\end{aligned} \tag{12}
$$

We now take the classical limit of the spin variable in order to define coherent states that are eigenstates of the bosonic operators a, b and c. On each point of the continuous chain we define a coherent state

$$
|\alpha(z)\,\beta(z)\,\gamma(z)\,\rangle = |\alpha(z)\rangle|\beta(z)\rangle|\gamma(z)\rangle \tag{13}
$$

with

$$
|\alpha(z)\rangle = \exp(-|\alpha|^2)\sum_{n=0}^{\infty}\frac{\alpha(z)^n}{\sqrt{n!}}\,|n\rangle \tag{14}
$$

$$
|\beta(z)\rangle = \exp(-|\beta|^2)\sum_{n=0}^{\infty}\frac{\beta(z)^n}{\sqrt{n!}}\,|n\rangle \tag{15}
$$

$$
|\gamma(z)\rangle = \exp(-|\gamma|^2)\sum_{n=0}^{\infty}\frac{\gamma(z)^n}{\sqrt{n!}}\,|n\rangle \tag{16}
$$

As the states $|\alpha\,\beta\,\gamma\rangle$ are eigenvectors of the operators a, b, c with eigenvalues $\alpha\,\beta\,\gamma$, respectively, it is possible to write down the coupled differential equations for this eigenvalues through bracketing the equations for the bosonic operators into and arbitrary coherent state. Note also that the meaning of the average of a derivative in our coherent state formalism is

$$\left\langle\alpha\left|\frac{\partial}{\partial\xi}a(\xi)\right|\alpha\right\rangle = \langle\,\alpha\,|\lim_{\Delta\xi\to 0}[a(\xi+\Delta\xi)\,-\,a(\xi)]|\alpha\rangle$$
$$= \lim_{\Delta\xi\to 0}(\alpha(\xi+\Delta\xi)\,-\,\alpha(\xi))\langle\alpha|\alpha\rangle$$
$$= \frac{\partial}{\partial\xi}\alpha. \tag{17}$$

So we obtain

$$\begin{aligned}\mathrm{i}\,\dot\alpha = {} & J(-\tfrac{3}{8}\alpha'' - \tfrac{1}{4}|\alpha|^2\alpha'' - \alpha'\beta'^*\beta - \tfrac{1}{4}\alpha\beta''^*\beta - \tfrac{1}{2}|\beta|^2\alpha'') \\ & + J(\tfrac{1}{2}\alpha|\beta'|^2 + \tfrac{1}{4}\alpha\beta^*\beta'' - \tfrac{1}{4}\alpha''^*\alpha^2 - \tfrac{1}{2}|\alpha'|^2\alpha) \\ & + \tfrac{1}{2}D\,(|\alpha|^2 - |\beta|^2 + \tfrac{1}{2})\,\alpha - \tfrac{1}{2}\,h\,\beta + \tfrac{1}{2}\lambda\langle q''\rangle 2S\alpha - \tfrac{3}{2}\lambda\langle q'\rangle\alpha'\end{aligned} \tag{18}$$

$$\begin{aligned}\mathrm{i}\,\dot\beta = {} & J(-\tfrac{3}{8}\beta'' - \tfrac{1}{4}|\beta|^2\beta'' - \beta'\alpha'^*\alpha - \tfrac{1}{4}\beta\alpha''^*\alpha - \tfrac{1}{2}|\alpha|^2\beta'') \\ & + J(\tfrac{1}{2}\beta|\alpha'|^2 + \tfrac{1}{4}\beta\alpha^*\alpha'' - \tfrac{1}{4}\beta''^*\beta^2 - \tfrac{1}{2}|\beta'|^2\beta) \\ & + \tfrac{1}{2}D\,(|\beta|^2 - |\alpha|^2 + \tfrac{1}{2})\,\beta - \tfrac{1}{2}\,h\,\alpha + \tfrac{1}{2}\lambda\langle q''\rangle 2S\beta - \tfrac{3}{2}\lambda\langle q'\rangle\beta'\end{aligned} \tag{19}$$

$$\begin{aligned}\mathrm{i}\,\dot\gamma = {} & -\frac{\omega_0}{2}(\gamma^* - \gamma) - \frac{k}{m\omega_0}(\gamma^{*\prime\prime} + \gamma'') + \frac{\lambda}{\sqrt{2m\omega_0}}[2S\ \mathrm{Re}\,(\alpha^*\alpha'' + \beta^*\beta'')] \\ & + \frac{2\lambda}{\sqrt{2m\omega_0}}\ \mathrm{Re}\,(\alpha^*\beta^*\beta'\alpha' + \alpha^*\beta^{*\prime}\beta\alpha' + \frac{1}{2}(\alpha^*\alpha^*\alpha'\alpha' + \beta^*\beta^*\beta'\beta')) \\ & + \frac{\lambda}{\sqrt{2m\omega_0}}[2S(|\alpha'|^2 + |\beta'|^2) + |\alpha|^2|\alpha'|^2 + |\beta|^2|\beta'|^2] \\ & + \frac{3\lambda}{2\sqrt{2m\omega_0}}[|\alpha'|^2 + |\beta'|^2 + \alpha^*\alpha'' + \beta^*\beta'']\end{aligned} \tag{20}$$

where we have made use of the following kinematics conditions associated with the Schwinger transformation:

$$\begin{aligned}|\alpha(z,t)|^2 - |\beta(z,t)|^2 &= 2\rho(z,t) \\ |\alpha(z,t)|^2 + |\beta(z,t)|^2 &= 2S\end{aligned} \tag{21}$$

where $\rho(z,t)$ accounts for the out of plane spin deviations.

Solving for α and β we get, to first order in ρ

$$\begin{aligned}\alpha(z,t) &= \sqrt{S}(1 + \rho(z,t)/2S)\exp[\mathrm{i}\theta_\alpha(z,t)] \\ \beta(z,t) &= \sqrt{S}(1 - \rho(z,t)/2S)\exp[\mathrm{i}\theta_\beta(z,t)]\end{aligned} \tag{22}$$

where $\theta_\alpha(z,t)$ and $\theta_\beta(z,t)$ are real variables.

From these equations we can form, after separating real and imaginary parts, the following coupled non-linear differential equations:

$$\begin{aligned}(\tfrac{3}{8}+S)J\rho'' - \tfrac{3}{16}J\rho(\theta_\beta'^2+\theta_\alpha'^2) + \tfrac{3}{8}JS(\theta_\beta'^2-\theta_\alpha'^2) - \tfrac{1}{2}\rho(\dot\theta_\beta+\dot\theta_\alpha) \\ + \tfrac{1}{2}JS\rho\,(\theta_\beta'-\theta_\alpha')^2 + S(\dot\theta_\beta-\dot\theta_\alpha) - [2(\tfrac{1}{8}+S)D+\tfrac{1}{2}h\cos(\theta_\beta-\theta_\alpha)]\rho \\ + \tfrac{3}{2}\lambda\,\langle q'\rangle\rho' - \lambda\,\langle q''\rangle\rho = 0 \end{aligned} \tag{23}$$

$$\begin{aligned}-2\dot\rho - \tfrac{3}{4}J\rho'(\theta_\beta'+\theta_\alpha') + 2S[(\tfrac{3}{8}+S)\,(\theta_\beta''-\theta_\alpha'') - (h/J)\sin(\theta_\beta-\theta_\alpha)] \\ - \tfrac{3}{8}J(\theta_\beta''+\theta_\alpha'')\rho + \tfrac{3}{2}\lambda\langle q'\rangle[(\theta_\beta'-\theta_\alpha') - (1/2S)(\theta_\beta'+\theta_\alpha')\rho] = 0\end{aligned} \tag{24}$$

$$\begin{aligned}J\rho'(\theta_\beta'-\theta_\alpha') - J(\theta_\beta''+\theta_\alpha'') + J\rho(\theta_\beta''-\theta_\alpha'') - 2\lambda\langle q'\rangle[(\theta_\beta'+\theta_\alpha') - \tfrac{1}{2}(\theta_\beta'-\theta_\alpha')\rho] \\ = 0\end{aligned} \tag{25}$$

$$\operatorname{Im}\dot\gamma = \frac{2k}{m\omega_0}\operatorname{Re}\gamma'' \tag{26}$$

$$\operatorname{Re}\dot\gamma = \omega_0\operatorname{Im}\gamma + \frac{3\lambda}{2\sqrt{2m\omega_0}}[S(\theta_\beta''+\theta_\alpha'') - (\theta_\beta'-\theta_\alpha')\rho' - (\theta_\beta''-\theta_\alpha'')\rho\,]. \tag{27}$$

Let us consider first the sine–Gordon limit, i.e. $\rho = 0$; in this case equations (23), (24) and (25) are

$$(\theta_\beta'+\theta_\alpha')_o = -\frac{8\dot\phi}{3J\phi'} \tag{28}$$

$$\phi'' - \frac{8h}{11J}\sin\phi + \frac{6}{11}\lambda\langle q'\rangle\phi' = 0 \tag{29}$$

$$\langle q'\rangle = 0 \tag{30}$$

and then no elastic mode is excited when the out of plane excursions are omitted from the theory. We have only sine–Gordon magnetic kinks.

To first order in ρ we get from (25) and (28)

$$(\theta_\beta''+\theta_\alpha'') = \rho'\phi' + \rho\phi'' + \frac{16\lambda}{3J}\langle q'\rangle\frac{\dot\phi}{\phi'}. \tag{31}$$

Substituting this in (27) and then combining with (26) results in

$$\langle\ddot q\rangle = \frac{2k}{m}\langle q''\rangle + \frac{8\lambda^2}{mw}[\langle\dot q'\rangle\dot\phi\phi' + \langle q'\rangle\ddot\phi\phi' - \langle q'\rangle\dot\phi\dot\phi'](\phi')^{-2}. \tag{32}$$

Assuming permanent form solutions for the magnetic degrees of freedom, i.e., taking $\phi(z,t) = \phi(z-vt)$, the last two terms cancel each other and $\dot\phi\,\phi'(\phi')^{-2}$ becomes $-v$ and the above equation takes the form

$$\langle\ddot q(z,t)\rangle = \frac{2k}{m}\langle q''(z,t)\rangle - \frac{8\lambda^2 v}{m\omega_0}\langle\dot q'(z,t)\rangle. \tag{33}$$

On the other hand, taking also $q(z,t) = q(z - v_e t)$ and defining $\xi \equiv z - vt$ we get, after straightforward calculations using equations (18), (19) and (23), the following equation for the out-of-plane excursions of the spins:

$$\rho''(\xi) + G(\xi)\rho(\xi) - \frac{\sqrt{3J}\lambda S\,\phi'(\xi)}{2C_0 L(\xi)}\langle q''(\xi + (v - v_e)t)\rangle + \frac{3\lambda}{2C_0}\langle q'(\xi + (v - v_e)t)\rangle\rho'(\xi) = Q(\xi) \qquad (34)$$

where the derivatives are with respect to ξ and we use the time as a parameter; we also define

$$G(\xi) = \frac{1}{JC_0}[F(\xi) - H(\xi) - \lambda\langle q''(\xi - (v - v_e)t)\rangle] \qquad (35)$$

$$F(\xi) = \frac{J}{2}\left(1 - \frac{3}{16S}\right)\phi'^2(\xi) + \frac{2v^2}{3JS} - 2D\left(1 + \frac{1}{8S}\right) - \frac{h}{2S}\cos\phi(\xi) \qquad (36)$$

$$H(\xi) = \frac{Jv\phi'^2(\xi)}{(3J)^{1/2}L(\xi)} \qquad (37)$$

$$L(\xi) = \left[h\cos\phi(\xi) - \frac{1}{2}D - \frac{3J}{16}\phi'^2(\xi) + \frac{4v^2}{3J}\left(1 + \frac{16S}{3}\right)\right]^{1/2} \qquad (38)$$

$$Q(\xi) = -\frac{1}{2C_0 J^{1/2}}\phi'(\xi)L(\xi) \qquad (39)$$

where $C_0 = (3/8S + 1)$ and $\phi(\xi) = \cos^{-1}(1 - 2\,\mathrm{sech}^2(\sqrt{h/C_0}\,\xi))$

Equation (33) is a typical wave equation that admits harmonic solutions of the form

$$\langle q(z,t)\rangle = q_0\exp[\mathrm{i}(k_0 z - \omega t)] \qquad (40)$$

with

$$\begin{aligned} \omega &= \frac{4\lambda^2 k_0 v}{m\omega_0}\left(1 \pm \sqrt{1 + \frac{m^2\omega_0^4}{16\lambda^4 v^2}}\right) = (v - v_e)k_0 \qquad v - v_e > 0 \\ \omega &= \frac{4\lambda^2 k_0 v}{m\omega_0}\left(1 \mp \sqrt{1 + \frac{m^2\omega_0^4}{16\lambda^4 v^2}}\right) = (v - v_e)k_0 \qquad v - v_e < 0. \end{aligned} \qquad (41)$$

Equation (41) links the phase velocity v_e of the elastic harmonic wave with the velocity v of the permanent form pulse representing ρ.

We proceed replacing this harmonic solution in equation (34) and performing a numerical integration taking $J = 1$ and $S = 1$. For some values of the parameters we have found solitary excursions out of the easy plane; one for each 2π sine–Gordon rotation around the easy axis. The initial conditions used were $\rho(-\infty) = \rho'(-\infty) = 0$ and we want the final conditions $\rho(+\infty) = \rho'(+\infty) = 0$. Regarding the velocity of the solitons there are two limiting cases in which these final conditions are not satisfied. First is the case of low v; here the solutions correspond to very large ρ that can even be divergent. In figure 1 we see that the role of λ and v is to decrease ρ. Using

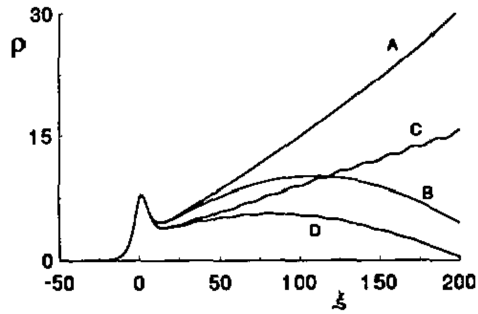
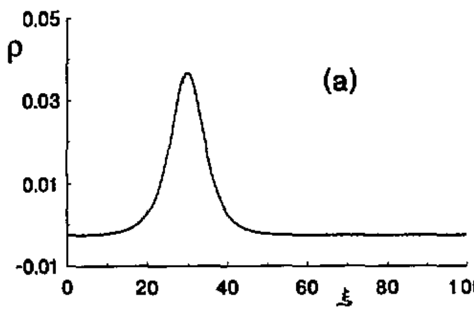


Figure 1. Out-of-easy-plane excursion $\rho(\xi, t=1)$ for two values of the velocity and two values of λ; curve A, $v=5$, $\lambda=0.001$; curve B, $v=13$, $\lambda=0.001$; curve C, $v=5$, $\lambda=30$; curve D, $v=13$, $\lambda=30$. The parameters are: $h=0.1$, $d\equiv D/J$=0.2, $m=k=20$.



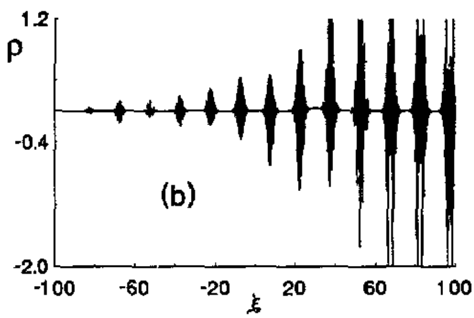


Figure 2. Out-of-easy-plane excursion $\rho(\xi, t=1)$ for two values of the velocity (a) $v=30$; (b) $v=60$. The parameters are: $\lambda=30$ $h=0.1$, $d\equiv D/J=0.2$, $m=k=20$.

this fact we get the small amplitude soliton presented in figure 2(a). In figure 3 we show the approximate lower value of the velocity below which the soliton amplitude is extremely large. The second limiting case correspond to larger values of v up to values above which the excursions diverge in high frequency back an forward oscillations as can be seen in figure 2(b); this effect has proved to be enhanced by the presence of the lattice harmonic oscillations.

We consider now as solution of equation (33) the pulse found in references [3] and [5] for the compressible ferromagnetic Heisenberg chain with uniaxial anisotropy

$$\langle q(\xi_e)\rangle = q_0 \tanh(k_0\xi_e)\operatorname{sech}(k_0\xi_e) \tag{42}$$

where $\xi_e = z + v_e t$.

Using this changing-sign pulse in (34) and performing a numerical integration we studied the effects of the lattice deformation on a traveling magnetic soliton. The moduli of the velocities are related by the equation (41) and we supposed the magnetic soliton traveling from the left and the lattice deformation from the right. The centres coincide at $t=0$. In figure 4 we present ρ for different times showing the effect of the collision with the elastic pulse. The magnetic field h modifies the deformation of the magnetic soliton due to the collision at $t=0$, in the form shown in figure 5; we observe that the field makes the magnetic soliton less perturbable by the elastic

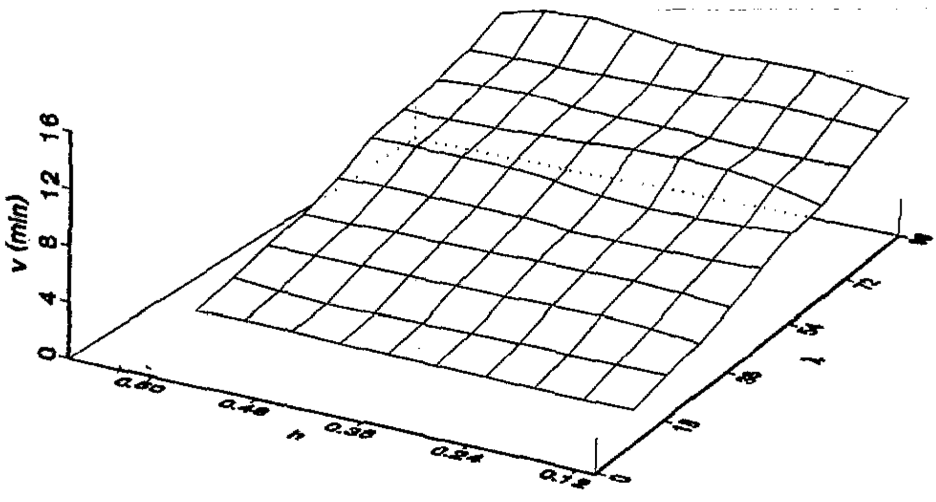


Figure 3. Dependence on λ and h of the minimum value of the velocity in order to have low amplitude solitons with $\rho(\pm\infty) = 0$ and $\rho'(\pm\infty) = 0$.

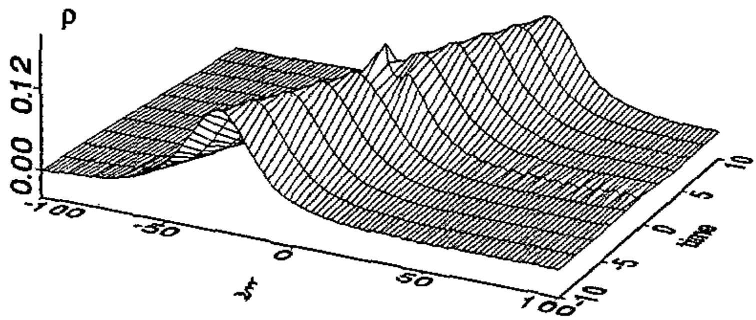


Figure 4. An out-of-easy-plane excursion $\rho(\xi, t)$ coming from the right encounters an elastic pulse coming from the left. The parameters are: $\lambda = 20$, $h = 0.01$, $d \equiv D/J = 0.2$, $m = k = 20$ and $v = 3$.

pulse. Finally, we present in figure 6 the dependence of the deformation, at $t = 0$, on the magnetostrictive parameter λ; as λ increases the deformation becomes more pronounced and irregular.

According to the results presented above we can recognize the existence of coupled magnetic and elastic non-linear excitations in the compressible Heisenberg chain with planar anisotropy. These modes are however not possible in the absence of out-of-plane excursions of the spins. Our analysis considers two situations that may be improved: it is a first order theory on the out-of-plane excursions and it considers only harmonic terms in the lattice Hamiltonian. To include nonharmonic elastic terms and take higher order terms in ρ seems to be the next step to be considered. The irregular oscillations observed for a large coupling between the magnetic and elastic modes suggest the idea of analyzing a discrete chain introducing a mapping and investigate further the spectral properties of the corresponding stochastic structures. Also the case of a compressible antiferromagnetic chain may be treatable with this formalism

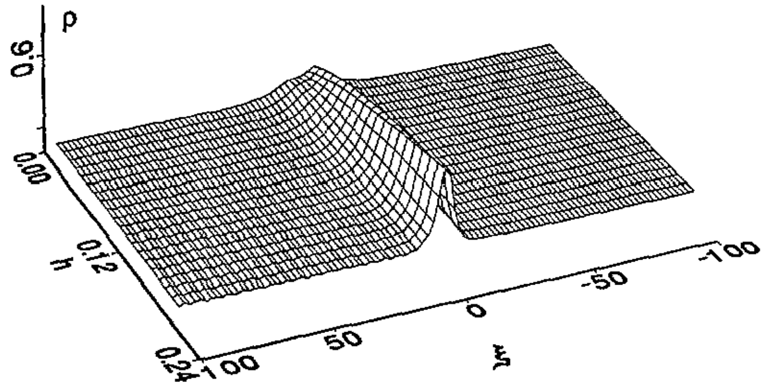


Figure 5. Effects of the in-plane external field h on the out-of-easy-plane excursion $\rho(\xi, t=0)$. The parameters are: $\lambda = 20$, $d \equiv D/J = 0.2$, $m = k = 20$ and $v = 3$.

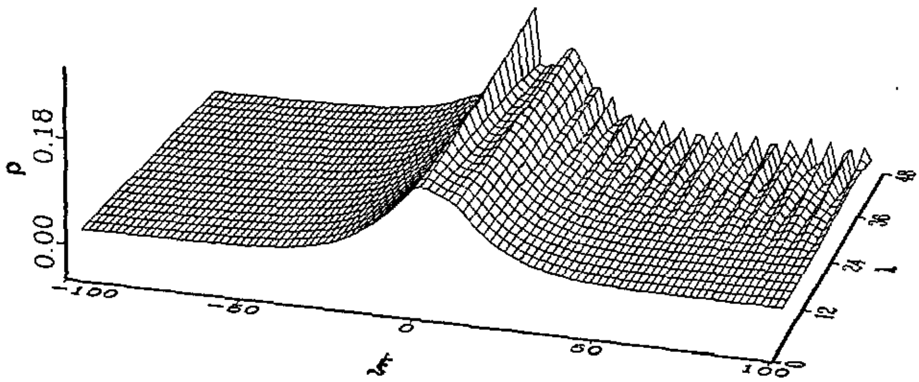


Figure 6. Effects of the magnetostrictive parameter λ on the out-of-easy-plane excursion $\rho(\xi, t=0)$. The parameters are: $h = 0.01$, $d \equiv D/J = 0.2$, $m = k = 20$ and $v = 3$.

to see the effects of the compressibility on the crossover fields [6].

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